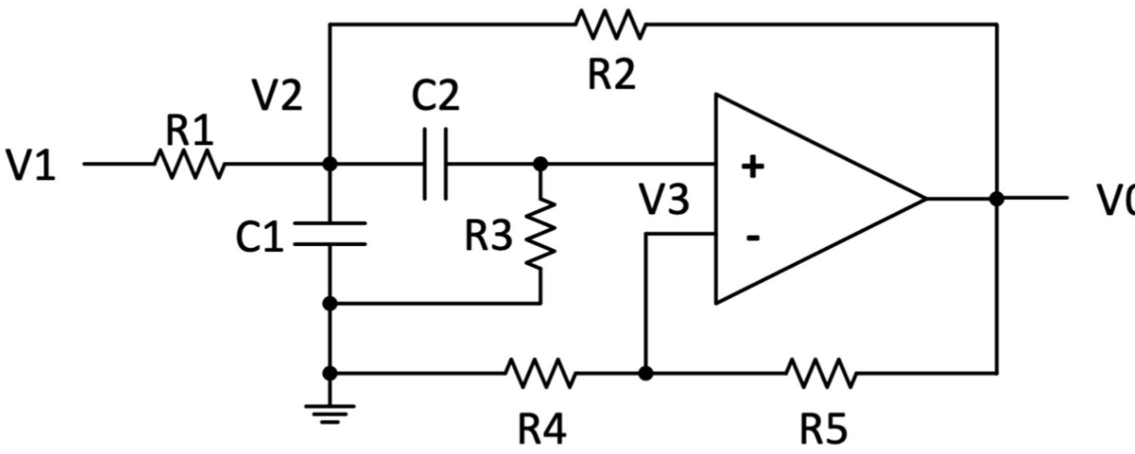
Q4, 1




At $V_-$ KCL:

$$0 = \frac{0 - V_3}{R_4} + \frac{V_0 - V_3}{R_5}$$

$$V_3\left(\frac{R_4 + R_5}{R_4}\right) = V_0 \qquad (i)$$

At $V_+$: KCL:

$$0 = V_2 sC_2 - V_3 sC_2 - \frac{V_3}{R_3}$$

$$V_3\left(\frac{1 + R_3 C_2 s}{R_3}\right) = V_2 s C_2$$

$$V_3 = V_2 \cdot \frac{sR_3C_2}{1 + sR_3C_2} \qquad (ii)$$

(i) -> (ii)

$$\therefore \quad V_2 = \frac{R_4}{R_4 + R_5} \cdot \frac{1 + sR_3C_2}{sR_3C_2}$$

At $V_2$: $$\frac{V_1 - V_2}{R_1} + \frac{V_0 - V_2}{R_2} + \frac{0 - V_2}{1/sC_1} + \frac{V_3 - V_2}{1/sC_2} = 0$$

$$\frac{V_1}{R_1} + \frac{V_0}{R_2} + V_3 sC_2 = V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + sC_2\right) \quad \text{(iii)}$$

(i) $\rightarrow$ (iii)

$\therefore$

$$\frac{V_1}{R_1} + \frac{V_0}{R_2} + V_0\frac{R_4}{R_4+R_5} \cdot sC_2 = \frac{R_4}{R_4+R_5} \cdot \frac{1+sR_3C_2}{sR_3C_2}\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + sC_2\right)$$

if $K = \dfrac{R_4 + R_5}{R_4}$

$$K\frac{sR_3C_2}{R_1}V_1 = V_0\left[(1+sR_3C_2)\left(\frac{1}{R_1} + \frac{1}{R_2} + s(C_1+C_2)\right) - K\frac{sR_3C_2}{R_2} - s^2R_3C_2^2\right]$$

$$\therefore \frac{V_0}{V_1} = \frac{s\dfrac{K}{R_1C_1}}{s^2 + s\left[\dfrac{1}{R_3}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) + \dfrac{1}{C_1}\left(\dfrac{1}{R_1} - \dfrac{R_5}{R_2R_4}\right)\right] + \dfrac{1}{R_3C_1C_2}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)}$$

where $K = 1 + \dfrac{R_5}{R_4}$

Q4.2: change the resistors of the summing circuit to create an inverting amplifier stage

Q4.3: $\frac{\zeta_1}{\zeta_2} = 0{,}01$ & $\omega_n = 160$ rad/s

$$\omega_n^2 = \frac{1}{R_1C_1R_2C_2} \quad (i)$$

$$\cancel{2}\zeta_1\omega_n = \frac{\cancel{2}}{R_1C_1}$$

$$\omega_n \zeta_1 = \frac{1}{R_1C_1} \quad ii)$$

$$2\zeta_2\omega_n = \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right) \quad (iii)$$

(ii) -> (iii) $$2\zeta_2\omega_n = \omega_n\zeta_1 + \frac{1}{R_2C_2}$$

$$(2\zeta_2 - \zeta_1)\omega_n = \frac{1}{R_2C_2} \quad (iv)$$

(iv) -> (i) $$\omega_n^2 = \omega_n^2\zeta_1(2\zeta_2 - \zeta_1)$$

but $\zeta_1 = 0{,}01\zeta_2$

$$\therefore 1 = 2\zeta_1(100\zeta_1) - \zeta_1^2$$

$$\therefore \zeta_1 = 0{,}0709$$

$$\zeta_2 = 7{,}09$$

from ii) $160 \cdot 0{,}0709 = \frac{1}{R_1 C_1}$

$\therefore$ choose $C_1 = 10\,\mu F$

$\rightarrow R_1 = 8815\ \Omega$

$\therefore$ choose $R_1 = 8{,}6\ k\Omega$ (E12)

from iv) $160(2 \cdot 7{,}09 - 0{,}0709) = \frac{1}{R_2 C_2}$

choose $C_2 = 1\ \mu F$

$\rightarrow R_2 = 442$

$\therefore$ choose $R_2 = 4{,}7\ k\Omega$ (E12)

These will result in a non-zero error from the specifications

Q4.2:

Top circuit:

At $V_-$: $$0 = \frac{V_0(1+sC_2R_2)}{R_2} + \frac{V_1(1+sC_1R_1)}{R_1}$$

$$\therefore \frac{V_0}{V_1} = -\frac{R_2}{R_1}\frac{(1+sC_1R_1)}{(1+sC_2R_2)}$$

Bottom:

$$V_- = V_0 \frac{\left(\frac{R_1}{1+sC_1R_1}\right)}{\frac{R_1}{1+sC_1R_1} + R_2}$$

$$V_- = V_0 \cdot \frac{R_1}{R_1 + R_2 + sC_1R_1R_2} \qquad (i)$$

$$V_- = V_0 \cdot \frac{1}{1 + \frac{R_2}{R_1} + sC_1R_2}$$

At $V_+$: $$V_+ = V_1 \cdot \frac{1/sC}{\frac{1}{sC} + R}$$

$$V_+ = V_1 \cdot \frac{1}{1 + sCR} \qquad (ii)$$

$$\therefore \quad \frac{V_1}{1+sCR} = \frac{V_0}{1+\frac{R_2}{R_1}+sC_1R_2}$$

$$\frac{V_0}{V_1} = \left(1+\frac{R_2}{R_1}+sC_1R_2\right) \cdot \frac{1}{1+sCR}$$

$$\frac{V_0}{V_1} = \left(1+\frac{R_2}{R_1}\right) \frac{1+\frac{sC_1R_1R_2}{R_1+R_2}}{1+sCR}$$